

Saturation-dependent anisotropy of unsaturated soils

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Abstract

The effect of saturation degree on hydraulic conductivity anisotropy in unsaturated soils is still an outstanding issue. This study investigates the impact of soil texture and soil bulk density on the degree of saturation-dependent anisotropy of layered soils by combining pedo-transfer function (PTF) results with the thin layer concept. The main objective is to examine how anisotropy characteristics are related to the relationships between hydraulic properties and the basic soil attributes such as texture and bulk density. The hydraulic parameters are related to the texture and bulk density based on the PTF results through linear regression. The results illustrate that the coupled dependence of the hydraulic parameters on the texture and bulk density is important to determine the anisotropic behavior and the inter-relationships of soil texture, bulk density, and hydraulic properties may cause very different saturation-dependent of unsaturated soils.

Key Words

Arithmetic mean, harmonic mean, anisotropy factor, capillary pressure, minimum anisotropy.

Introduction

Large scale heterogeneous soils often demonstrate different moisture spreading and solute transport patterns at different saturation degrees (or capillary pressure head levels). While saturation-dependent anisotropy has been recognized for a long time (e.g., Zaslavsky and Sinai, 1981; Stephens and Heerman, 1988), it has not been fully understood. Mualem (1984) proposed a conceptual model to quantify the capillary pressure-dependent anisotropy by assuming that soils consist of many thin layers. A similar concept was extended to consider effects of bulk density variations within a particular soil type (Assouline and Or, 2006). Other approaches have also been proposed to study the soil anisotropy behavior. Zhang *et al.* (2003) proposed a tensorial connectivity-tortuosity concept to describe the unsaturated soil hydraulic conductivities. McCord *et al.* (1991) described a series of soil water tracer experiments and approaches to numerically model the flow behavior observed in field experiments. These experimental and numerical results provided strong supporting evidence for a variable, saturation-dependent anisotropy in the hydraulic conductivity of an unsaturated medium. Green and Freyberg (1995) calculated the capillary pressure-dependent anisotropy under conditions of large-scale gravity drainage. Ursino *et al.* (2000) used Miller similitude with different pore-scale geometries of the basic element to model macroscopic flow and transport behavior. Their results demonstrated that the geometry of the microstructure could lead to anisotropic behavior at larger scale even if the system is characterized by an isotropic correlation structure. Khaleel *et al.* (2002) used a unit-mean-gradient approach to derive upscaled hydraulic properties for flow parallel and perpendicular to bedding by simulating steady gravity drainage conditions for a series of applied infiltration rates of relatively dry conditions in coarse-textured sediments. In this study, we investigate unsaturated soil anisotropy that arises from a combination of soil texture and bulk density variations and the pedo-transfer function (PTF) results of soil hydraulic conductivities. PTFs transform basic soil properties such as texture and bulk density into water retention and unsaturated hydraulic conductivity. The main objective is to improve the fundamental understanding of various saturation-dependent anisotropy behaviors. Specifically, we examine how different inter-relationships of soil texture, bulk density and hydraulic properties may induce different anisotropy characteristics.

Methods

Hydraulic properties in relation to texture and bulk density

Van Genuchten (1980) combined the soil water retention function with the statistical pore-size distribution model and obtained the following hydraulic property functions,

$$Se = [1 + (\alpha h)^n]^{-m} \quad (1)$$

$$K = K_s [1 - (1 - Se^{1/m})^m]^2 \quad (2)$$

where $Se = (\theta - \theta_r) / (\theta_s - \theta_r)$ is the effective degree of saturation, θ is the volumetric water content, θ_r is the residual volumetric water content, θ_s is the saturated volumetric water content, h is the capillary pressure

head, K is the hydraulic conductivity, K_S is the saturated hydraulic conductivity; α , m and n are empirical hydraulic shape parameters, and $m=1-1/n$. Based on the results of the van Genuchten hydraulic parameters in relation to texture (using mean grain diameter d as a surrogate) and the bulk density established by the neural network based PTFs (Schaap and Leij, 1998) (see Table 1), we perform regression analyses to establish empirical linear relationships, which relate hydraulic properties to the two main indicators, the grain diameter d and the bulk density ρ .

Table 1: Mean soil grain diameters, average hydraulic parameters and bulk density values for the soil textural classes (Hydraulic parameters values are from Schaap and Leij (1998))

| Class | d (mm) | ρ (g/cm ³) | $\log(\alpha)$ ($\log(1/\text{cm})$) | $\log(n)$ | $\log(K_S)$ ($\log(\text{cm}/d)$) |
|-------------------|----------|-----------------------------|---|-----------|--|
| Sand | 0.9249 | 1.53 | -1.45 | 0.50 | 2.81 |
| Loamy Sand | 0.8245 | 1.52 | -1.46 | 0.24 | 2.02 |
| Loam | 0.4211 | 1.37 | -1.95 | 0.17 | 1.08 |
| Sandy Loam | 0.6222 | 1.46 | -1.57 | 0.16 | 1.58 |
| Silt Loam | 0.2214 | 1.28 | -2.30 | 0.22 | 1.26 |
| Sandy Clayey Loam | 0.6197 | 1.57 | -1.68 | 0.12 | 1.12 |
| Silty Clayey Loam | 0.1177 | 1.32 | -2.08 | 0.18 | 1.05 |
| Clay Loam | 0.4186 | 1.42 | -1.80 | 0.15 | 0.91 |
| Silt | 0.0747 | 1.33 | -2.18 | 0.22 | 1.64 |
| Clay | 0.2114 | 1.39 | -1.82 | 0.10 | 1.17 |
| Sandy Clay | 0.51601 | 1.59 | -1.48 | 0.08 | 1.06 |
| Silty Clay | 0.11644 | 1.36 | -1.79 | 0.12 | 0.98 |

From the linear regression analyses, it was found that $\log(n)$ is poorly correlated to either d or ρ . Hills *et al.* (1992) showed that the variability of soil hydraulic characteristics could be adequately modeled using a variable van Genuchten α with a deterministic van Genuchten n . Therefore, we use the mean value of n in this study. $\log(\alpha)$ is best correlated to ρ , and is also fairly correlated to d . The best linear regression is:

$$\log(\alpha) = 0.262d + 1.887\rho - 4.603 \quad (3)$$

where α is in (1/cm), ρ is in (g/cm³) and d in (mm). The correlation coefficient for this empirical regression relationship is 0.91.

$\log(K_S)$ is poorly correlated to ρ , and is fairly correlated to d . The best linear regression relationship is:

$$\log(K_S) = 1.272d + 0.851 \quad (4)$$

where K_S is in (cm/day). The correlation coefficient for this regression relationship is 0.66.

These empirical linear regression relationships are used to develop anisotropy models in the following section. While other non-linear regression relationships or more complicated relationships can also be easily incorporated, we use these linear regression relationships for the sake of simplicity because our goal is to focus on whether and how the inter-relationships of soil texture, bulk density, and hydraulic properties may affect the anisotropy behaviors of layered unsaturated soils.

Anisotropy model

We consider a soil consisting of a large number of thin, but distinguishable layers of different texture (as indicated by d) and the bulk density ρ . Each layer is characterized by its own van Genuchten hydraulic conductivity function, $K(\text{Se}, K_S, \alpha, n)$. Since the van Genuchten parameters have been related to d , and ρ as described in the previous section, the hydraulic conductivity can now be written in a general form of $K(\text{Se}, d, \rho)$. The layered formation is expressed in terms of a joint probability density function, $f(d, \rho)$, of the grain diameter d and the bulk density ρ . Parallel to the layering, the hydraulic conductivity $K_H(\text{Se})$, is described by the arithmetic mean of $K(\text{Se}, d, \rho)$ of the layers (Mualem, 1984; Assouline and Or, 2006),

$$K_H(\text{Se}) = \iint K(\text{Se}, d, \rho) f(d, \rho) d d d \rho \quad (5)$$

The hydraulic conductivity perpendicular to the layers, $K_N(\text{Se})$, is described as the harmonic mean of $K(\text{Se}, d, \rho)$ of the layers, which is expressed as follows,

$$K_N(\text{Se}) = \left[\iint \frac{f(d, \rho)}{K(\text{Se}, d, \rho)} d d d \rho \right]^{-1} \quad (6)$$

The anisotropy factor, A , can then be expressed as the ratio of the hydraulic conductivities in the parallel and perpendicular directions,

$$A(\text{Se}) = K_H(\text{Se})/K_N(\text{Se}) \quad (7)$$

For simplicity, we use uniform distributions to describe the probability distributions of both d and ρ , although other distributions could also be incorporated. Based on the probability density functions and the established regression relationships, we can calculate K_H (hydraulic conductivity parallel to the layering), K_N (hydraulic conductivity perpendicular to the layering), and A (anisotropy factor) as functions of the effective saturation degree Se from Eqns. (5) through (7), which can also be related to the capillary pressure head h .

Results and Discussion

Since K_S is fairly correlated only to the texture (with d as a surrogate), we first consider the case when K_S is related to d and α is related to both d and ρ through the regression relationships shown in Eqn. (4) and Eqn. (3) respectively. Figure 1 shows the relationships between the anisotropy factor A , and h as well the saturation degree Se for various combinations of d and ρ ranges and $n=1.59$. Since K_S is not as good as α in terms of correlation to either d and ρ , we also investigate the scenario that a simple constant K_S value equal to the mean value is used and α is related to both d and ρ . Figure 2 shows the relationships between the anisotropy factor A , and h as well Se for this scenario under otherwise same conditions as in Figure 1.

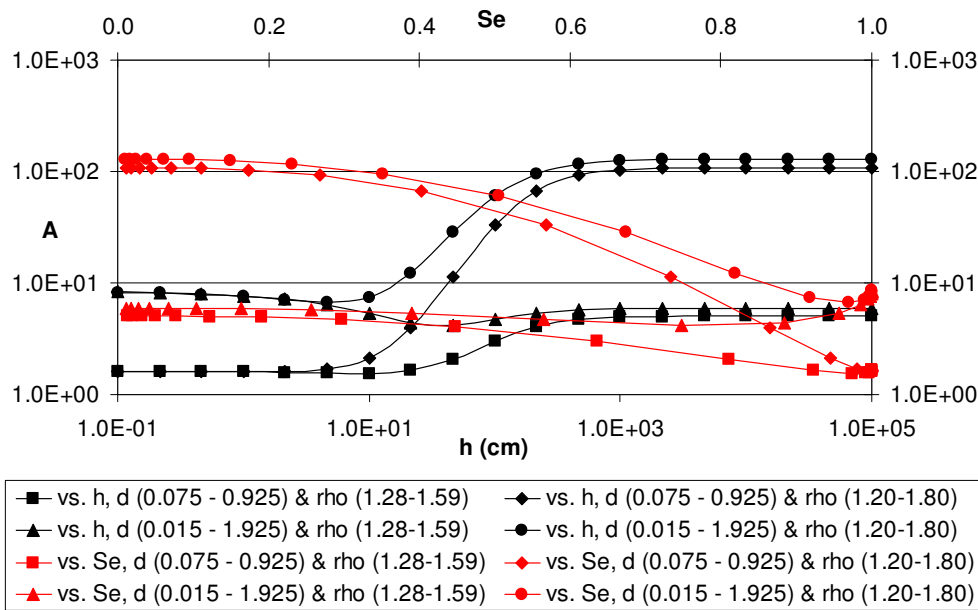


Figure 1. Relationships between the anisotropy factor A and the saturation degree Se as well as the capillary pressure head h . K_S is related to d and α is related to both d and ρ through regression relationship for various combinations of d and ρ ranges. $n=1.59$.

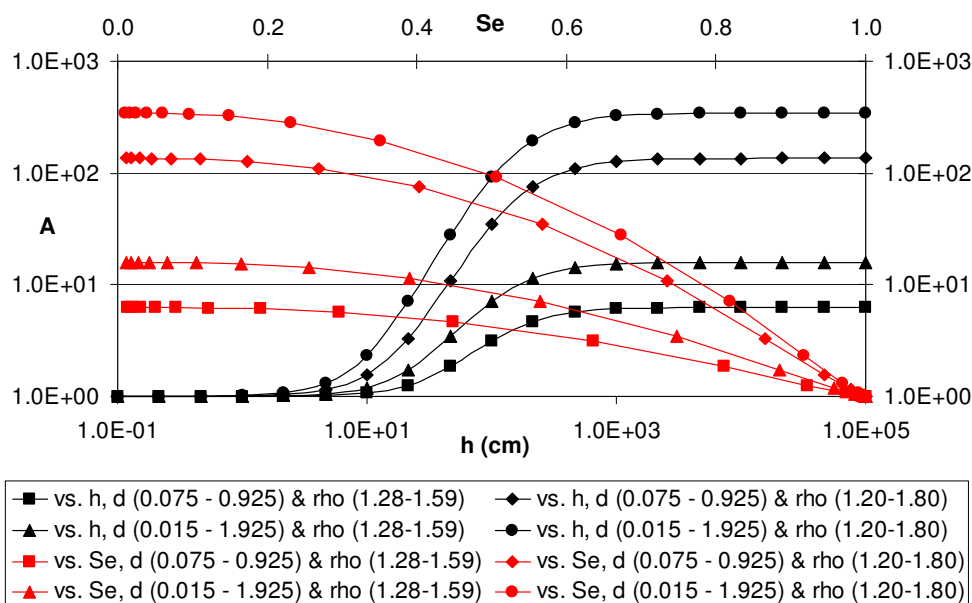


Figure 2. Relationships between the anisotropy factor A and the saturation degree Se as well as the capillary pressure head h . K_S is constant equal to the mean and α is related to both d and ρ through regression relationship for various combinations of d and ρ ranges. $n = 1.59$.

Results shown in Figures 1 and 2 indicate that anisotropy typically increases when the saturation degree decreases (or the capillary pressure head increases). When the grain diameter range is large (i.e., the heterogeneity is strong) and when K_S is related to d , and α is related to both d and ρ , a feature that the anisotropy factor A exhibits a minimum value at certain capillary pressure head (i.e., a non-monotonic relationship) is observed (black triangle symbolized curve in Figure 1). Many previous studies (McCord *et al.* 1991; Green and Freyberg, 1995; Assouline and Or, 2006) reported that the soil anisotropy first decreases as the capillary pressure increases and then increase as the capillary pressure further increases (i.e., the anisotropy reaches a minimum at a certain capillary pressure level). Other studies (e.g., Ursino *et al.* 2000; Khaleel *et al.* 2002) found that the soil anisotropy increases monotonically with increasing capillary pressure. This study illustrates that the anisotropy reaches a minimum only when both K_S and α are variables across the soil layers and the heterogeneity of soil attributes across layers is large. For other conditions, the anisotropy is found to increase monotonically with the decreasing saturation degree (or increasing capillary pressure head).

Conclusion

The key conclusions from this study include: 1) the coupled dependence of the hydraulic parameters on the texture and bulk density is important to determine the anisotropic behavior of unsaturated soils, and 2) the inter-relationships of soil texture, bulk density, and hydraulic properties may cause very different anisotropy behaviors of layered unsaturated soils.

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